

# Can large fermion chemical potentials suppress the electroweak phase transition?

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## Abstract

We calculate the critical temperature ( $T_c$ ) of the electroweak phase transition in the minimal standard model considering simultaneously temperature ( $T$ ) and fermion chemical potential ( $\mu_f$ ) effects over the effective potential. The calculation is performed in the one-loop approximation to the effective potential at non-zero temperature using the real time formalism of the thermal field theory. We show that it exists a fermion chemical potential critical value ( $\mu_f^c$ ) for which the Higgs boson condensate vanishes at  $T = 0$ . If  $T$  and  $\mu_f$  effects are considered simultaneously, it is shown that for  $\mu_f \geq \mu_f^c$  then  $T_c^2 \leq 0$ , implying that the electroweak phase transition might not take place.

*Keywords:* Minimal standard model, Spontaneous symmetry breaking, Electroweak phase transition, Critical temperature, Chemical potentials.

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# 1 Introduction

The original idea that spontaneously broken gauge symmetries might be restored at high temperatures for elementary particles systems in thermodynamic equilibrium was first presented by Kirzhnits and Linde [1]. They developed this idea for the case of field theories with global gauge symmetries, while for theories with local gauge symmetries it was done by Dolan and Jackiw [2] and Weinberg [3]. Later Linde found that the Higgs boson condensate, responsible for the spontaneous symmetry breaking in gauge theories, depends not only on temperature ( $T$ ) but also on fermion chemical potentials ( $\mu_f$ ) [4, 5]. In reference [4] Linde showed that in most gauge theories with neutral currents an increase of  $\mu_f$  leads to an increase of symmetry breaking. This behaviour was shown explicitly for the abelian Higgs model extended by the inclusion of fermions. For the Minimal Standard Model (MSM) it was shown [4] that the electroweak symmetry may not be restored in the early Universe if an excess of neutrinos over anti-neutrinos is sufficiently large at present. It should be mentioned that Linde's results [4] were obtained in the tree level approximation.

On the other hand, it has been established that the MSM is discarded as the framework of a possible mechanism of baryogenesis that gives a satisfactory explanation of the current Baryon Asymmetry of the Universe (BAU) [6]. This needling fact is due to the smallness of CP violation effects when fermion damping rates are included [6]-[9]. Until now it has not been presented a convincing mechanism of baryogenesis at temperatures below the critical temperature ( $T_c$ ) of the electroweak phase transition in the MSM [10]. However it is plausible that the BAU might have been generated at a  $T$  above  $T_c$  through a mechanism of baryogenesis in which  $B - L$  symmetry is conserved [11]. By this reason we consider a scenario in which  $B - L$  is conserved and the BAU has been generated at a  $T$  above  $T_c$  [12]. In this scenario the number of particles can be slightly larger than the number of anti-particles implying non-vanishing fermion chemical potentials ( $\mu_f \neq 0$ ). Consequently the electroweak phase transition might take place in a plasma characterized by an excess of particles over anti-particles, i.e.  $\mu_f \neq 0$ .

The main goal of this work is to calculate the  $T_c$  of the electroweak phase transition in the MSM for a plasma characterized by  $\mu_f \neq 0$ . This calculation is performed in the one-loop approximation to the effective potential at finite temperature, working in the the real time formalism of the thermal field theory [13]-[14] and using the Feynman gauge. We first calculate the  $T$  and  $\mu_f$  effects over the effective potential and then we obtain the Higgs boson condensate dependence on  $T$  and  $\mu_f$ . We show that at  $T = 0$  it exists a chemical potential critical value  $\mu_f^c$  for which the electroweak symmetry is restored. If  $T$  and  $\mu_f$  effects are simultaneously considered it is shown that for  $\mu_f \geq \mu_f^c$  then  $T_c^2 \leq 0$ , implying that the electroweak phase transition does not take place in this scenario. In the limit  $\mu_f = 0$  we reproduce the expression for

$T_c$  published in [15].

The one-loop approximation to the effective potential at finite temperature has been calculated using the method shown at [15]. Before performing our calculation for the MSM, which will be done in section 3, we first consider the abelian Higgs model extended by the inclusion of fermions in section 2. The study of this toy model allows to investigate the dependence of the Higgs boson condensate on  $T$  and  $\mu_f$ , and to calculate the  $T_c$  for the case of an abelian gauge theory. Finally our conclusions are presented in section 4.

## 2 The Abelian Higgs Model

The abelian Higgs Model extended by the inclusion of a fermion field  $\psi$  and a Yukawa coupling term is given by the following Lagrangian:

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + (\partial_\mu + ieA_\mu)\bar{\phi}(\partial_\mu - ieA_\mu)\phi - \lambda\left(\bar{\phi}\phi - \frac{b^2}{2\lambda}\right)^2 + \bar{\psi}i\gamma_\mu(\partial_\mu - ieA_\mu)\psi + Y_f\bar{\psi}\phi\psi + Y_f\bar{\psi}\bar{\phi}\psi, \quad (2.1)$$

where the complex scalar field is  $\phi = \frac{1}{\sqrt{2}}(H + i\eta)$  and  $\bar{\phi}$  is its hermitic conjugate field, being  $H$  the Higgs boson field,  $\eta$  the Goldstone boson field and  $Y_f$  the Yukawa coupling constant between  $\psi$  and  $\phi$ . We observe that the potential at zero temperature  $V(\phi, \bar{\phi}) = \lambda(\bar{\phi}\phi - \frac{b^2}{2\lambda})^2$  has a minimum value located at  $\langle \phi\bar{\phi} \rangle = H^2 + \eta^2 = \nu_o^2$ , where  $\nu_o^2 = \frac{b^2}{2\lambda}$ . The abelian gauge symmetry of the model is spontaneously broken by the choice of a definite ground state, for instance  $H = \nu_o$  and  $\eta = 0$ . This mechanism of spontaneous symmetry breaking, usually called Higgs mechanism, acts as the generator for the gauge boson and fermion masses of the model described by (2.1) in the framework of quantum field theory at zero temperature.

For the statistical model in thermodynamic equilibrium, with the same particle content and interactions as described by Lagrangian (2.1), it is well known that for  $\mu_f = 0$  the gauge symmetry of the model at  $T = 0$  is spontaneously broken due to the appearance of the Higgs boson condensate, i.e.  $\langle \phi\bar{\phi} \rangle = \nu_o^2$ . This statistical system presents a phase transition at a critical temperature ( $T_c$ ): for temperatures above  $T_c$  the system is in the symmetric phase,  $\nu = 0$ , while for temperatures below  $T_c$  it is in the broken symmetry phase,  $\nu \neq 0$ . However, as we will show later, this phase transition might not occur if  $T$  and  $\mu_f$  effects are simultaneously considered as  $\langle \phi\bar{\phi} \rangle$  vanishes for  $\mu_f \geq \mu_f^c$  at  $T \geq 0$ .

In this section we consider a thermal medium constituted by fermions, anti-fermions, Higgs bosons and gauge bosons, characterized by  $\mu_f \neq 0$ , where  $\mu_f$  is the chemical potential associated to an excess of fermions over anti-fermions in the

plasma. At finite temperature and density the Feynman rules for vertices are the same as those at  $T = 0$  and  $\mu_f = 0$ , and the propagators in the Feynman gauge for massless gauge boson  $D_{\mu\nu}(p)$ , massless scalars  $D(p)$  and massless fermions  $S(p)$  are [16]:

$$D_{\mu\nu}(p) = -g_{\mu\nu} \left[ \frac{1}{p^2 + i\epsilon} - i\Gamma_b(p) \right], \quad (2.2)$$

$$D(p) = \frac{1}{p^2 + i\epsilon} - i\Gamma_b(p), \quad (2.3)$$

$$S(p) = \frac{\not{p}}{p^2 + i\epsilon} + i\Gamma_f(p), \quad (2.4)$$

where  $p$  is the particle four-momentum. The plasma temperature  $T$  is introduced through the  $\Gamma_b(p)$  and  $\Gamma_f(p)$  functions given by:

$$\Gamma_b(p) = 2\pi\delta(p^2)n_b(p), \quad (2.5)$$

$$\Gamma_f(p) = 2\pi\delta(p^2)n_f(p), \quad (2.6)$$

with

$$n_b(p) = \frac{1}{e^{(p \cdot u)/T} - 1}, \quad (2.7)$$

$$n_f(p) = \theta(p \cdot u)n_f^-(p) + \theta(-p \cdot u)n_f^+(p) \quad (2.8)$$

where  $n_b(p)$  is the Bose-Einstein distribution function. The Fermi-Dirac distribution functions for fermions  $n_f^-(p)$  and for anti-fermions  $n_f^+(p)$  are:

$$n_f^\pm(p) = \frac{1}{e^{(p \cdot u \pm \mu_f)/T} + 1}. \quad (2.9)$$

In the distribution functions (2.7) and (2.8)  $u^\alpha$  is the four-velocity of the center-mass frame of the dense plasma with  $u^\alpha u_\alpha = 1$ .

If fermion density effects are not considered the change of  $\langle \phi \bar{\phi} \rangle$ , from  $\nu_o^2$  at  $T = 0$  to zero at  $T = T_c$ , is due to the temperature corrections to the effective potential of the complex scalar field. Now we calculate this effective potential considering simultaneously  $T$  and  $\mu_f$  effects following the same procedure as shown in [15]. We first calculate the polarization operator  $\Pi_\beta^\phi$  of the complex scalar field  $\phi$  at zero external momentum:

$$\Pi_\beta^\phi(0) = \left[ \frac{\partial^2 V_\beta(\phi, \bar{\phi})}{\partial \phi \partial \bar{\phi}} \right]_{\phi=0}. \quad (2.10)$$

The one-loop diagrams which contribute to  $\Pi_\beta^\phi$  are shown in Fig. 1. The contributes of these diagrams to  $\Pi_\beta^\phi$  at leading order in  $T^2$  and  $\mu_f^2$  are:

$$\Pi_{(a)}(0) = \frac{1}{3} \lambda T^2, \quad (2.11)$$

$$\Pi_{(b)}(0) = \frac{1}{3} e^2 T^2, \quad (2.12)$$

$$\Pi_{(c)}(0) = -\frac{1}{12} e^2 T^2, \quad (2.13)$$

$$\Pi_{(d)}(0) = \frac{Y_f^2}{2} \left[ \frac{T^2}{3} + \frac{\mu_f^2}{\pi^2} \right]. \quad (2.14)$$

We observe that the  $\mu_f$  effects over the effective potential at finite temperature is due to the contribute of diagram (d) shown in Fig. 1., which includes a fermion propagator. The full effective potential is:

$$V(\phi, \bar{\phi}) + V_\beta(\phi, \bar{\phi}) = \lambda(\phi\bar{\phi} - \nu_o^2)^2 + \left[ \frac{1}{12}(4\lambda + 3e^2 + 2Y_f^2)T^2 + Y_f^2 \frac{\mu_f^2}{2\pi^2} \right] \phi\bar{\phi}. \quad (2.15)$$

The Higgs boson condensate dependence on  $T$  and  $\mu_f$  is given by:

$$\nu^2(T, \mu_f) = \nu_o^2 - \frac{1}{6} \left( 1 + \frac{3e^2}{4\lambda} + \frac{Y_f^2}{2\lambda} \right) T^2 - \frac{1}{4\pi^2} \frac{Y_f^2}{\lambda} \mu_f^2. \quad (2.16)$$

For the purpose of reproducing known results we put  $T = 0$  in (2.16) and we obtain:

$$\nu^2(0, \mu_f) = \nu_o^2 - \frac{1}{4\pi^2} \frac{Y_f^2}{\lambda} \mu_f^2. \quad (2.17)$$

As it is possible to observe in (2.17)  $\nu(0, \mu_f)$  turns to be zero at a fermion chemical potential critical value  $\mu_f^c$  given by:

$$\mu_f^c = \sqrt{2\pi} \frac{b}{Y_f}. \quad (2.18)$$

This result is in accordance with equation (5.3) of reference [5] implying that, if temperature effects are not included, a second order phase transition with symmetry restoration takes place [5]. The Higgs boson condensate (2.16) turns to be zero at:

$$T = T_c = 2\nu_o \sqrt{\frac{6\lambda - 3(Y_f^2 \mu_f^2 / 2\pi^2 \nu_o^2)}{4\lambda + 3e^2 + 2Y_f^2}}. \quad (2.19)$$

We observe that if fermions are not introduced in the Higgs model Lagrangian (2.1), i.e.  $Y_f = 0$  in (2.16), then (2.19) reduces to:

$$T_c = 2\nu_o \sqrt{6\lambda / (4\lambda + 3e^2)}, \quad (2.20)$$

in agreement with [15]. Expression (2.19) is very interesting because it shows the effect of  $\mu_f$  on the  $T_c$  value. We note from (2.19) that for:

$$\mu_f = \mu_f^c = \pi\nu_o \frac{m_H}{m_f}, \quad (2.21)$$

where  $m_H^2 = 2\lambda\nu_o^2$  and  $m_f^2 = Y_f^2\nu_o^2/2$ , then  $T_c = 0$ . This means that if we consider simultaneously  $T$  and  $\mu_f$  effects, and if  $\mu_f \geq \mu_f^c$ , then  $T_c^2 \leq 0$ . This implies that in the above scenario a phase transition can not take place at any temperature. In other words if  $\mu_f$  is sufficiently large the gauge symmetry of the Higgs model extended by the inclusion of fermions is not spontaneously broken for  $T \geq 0$ .

### 3 The Minimal Standard Model

In this section we will calculate the  $T_c$  of the electroweak phase transition in the MSM. The scenario considered corresponds to an electroweak plasma in thermodynamical equilibrium characterized by unknown non-vanishing fermion chemical potentials. We consider for quarks  $\mu_u \neq \mu_d \neq \mu_c \neq \dots \neq 0$  and for charged leptons  $\mu_e \neq \mu_\mu \neq \mu_\tau \neq 0$ . Following a similar procedure as the one of section 2 we obtain the effective potential at non-zero temperature:

$$\begin{aligned} V_\beta(\phi, \bar{\phi}) = & \left[ 2\lambda + \frac{3g^2}{4} + \frac{g'^2}{4} + \sum_q Y_q^2 + \frac{1}{3} \sum_l Y_l^2 \right] \frac{T^2}{4} \phi \bar{\phi} \\ & + \left[ \frac{3}{4\pi^2} \sum_q Y_q^2 \mu_q^2 + \frac{1}{4\pi^2} \sum_l Y_l^2 \mu_l^2 \right] \phi \bar{\phi}, \end{aligned} \quad (3.1)$$

where the only novelty is the contribute from the different fermion species through the Feynman diagrams as the one of figure (1d). With this result the Higgs boson condensate depends on  $T$  and  $\mu_f$  as:

$$\begin{aligned} \nu(T, \mu_f)^2 = \nu_o^2 - \left[ 1 + \frac{3g^2}{8\lambda} + \frac{g'^2}{8\lambda} + \frac{1}{2\lambda} \sum_q Y_q^2 + \frac{1}{6\lambda} \sum_l Y_l^2 \right] \frac{T^2}{4} \\ - \frac{1}{8\pi^2\lambda} \left[ 3 \sum_q Y_q^2 \mu_q^2 + \sum_l Y_l^2 \mu_l^2 \right]. \end{aligned} \quad (3. 2)$$

The electroweak phase transition occurs at

$$T = T_c(\mu_f) = 2\nu_o \frac{\left[ 1 - \frac{3}{2\pi^2} \sum_q \frac{m_q^2}{m_H^2} \frac{\mu_q^2}{\nu_o^2} - \frac{1}{2\pi^2} \sum_l \frac{m_l^2}{m_H^2} \frac{\mu_l^2}{\nu_o^2} \right]^{1/2}}{\left[ 1 + \frac{3g^2}{8\lambda} + \frac{g'^2}{8\lambda} + 2 \sum_q \frac{m_q^2}{m_H^2} + \frac{2}{3} \sum_l \frac{m_l^2}{m_H^2} \right]^{1/2}}, \quad (3. 3)$$

where we have used the notation  $m_f^2 = Y_f^2 \nu_o^2 / 2$  and  $m_H^2 = 2\lambda \nu_o^2$ . We observe that if we do not consider fermion contributes to the effective potential, i.e.  $m_f = 0$  in (3.3), we obtain:

$$T_c = 2\nu_o \left[ 1 + \frac{3g^2}{8\lambda} + \frac{g'^2}{8\lambda} \right]^{-1/2}, \quad (3. 4)$$

in agreement with [17]. If we put  $\mu_f = 0$  in (3.3), we obtain:

$$T_c(0) = 2\nu_o \left[ 1 + \frac{3g^2}{8\lambda} + \frac{g'^2}{8\lambda} + 2 \sum_q \frac{m_q^2}{m_H^2} + \frac{2}{3} \sum_l \frac{m_l^2}{m_H^2} \right]^{-1/2}, \quad (3. 5)$$

in accordance with [15].

It is clear from (3.3) that if:

$$\frac{3}{2\pi^2} \sum_q \frac{m_q^2}{m_H^2} \frac{\mu_q^2}{\nu_o^2} + \frac{1}{2\pi^2} \sum_l \frac{m_l^2}{m_H^2} \frac{\mu_l^2}{\nu_o^2} = 1, \quad (3. 6)$$

then  $T_c = 0$ . The contributes to (3.3) coming from the different fermion species are proportional to  $m_f^2/m_H^2$ . The experimental lower bound on the Higgs boson mass,

$M_H = 107$  GeV [18], allows to conclude that the main contribute in (3.3) is due to the top quark mass ( $m_t$ ), and it is possible to write in good approximation (3.6) as:

$$\frac{3}{2\pi^2} \frac{m_t^2}{m_H^2} \frac{\mu_t^2}{\nu_o^2} \sim 1. \quad (3.7)$$

Taking  $m_t = 174$  GeV,  $\nu_o = 246$  GeV and  $m_H = 110$  GeV in (3.7), we obtain a chemical potential critical value:

$$\mu_t^c \sim 400 \text{ GeV}. \quad (3.8)$$

From (3.3) it can be stated that if  $\mu_t \geq \mu_t^c$  then  $T_c^2 \leq 0$ , therefore the electroweak phase transition can not take place in the above scenario.

For the case  $\mu_t < \mu_t^c$  we observe from (3.3) that  $T_c(\mu_t) < T_c(0)$ , being  $T_c(0) \sim 100$  GeV the critical temperature value of the electroweak phase transition for the case  $\mu_t = 0$ . The  $T_c(0)$  value can be obtained from (3.5) taking the same inputs as in (3.8). We note that the  $\mu_f$  values associated to the different fermion species of the MSM in the expression (3.3) are unknown parameters.

## 4 Conclusions

We have consider a  $B - L$  conserving thermal medium in thermodynamic equilibrium in which the Baryon Asymmetry of the Universe (BAU) has been generated at a temperature  $T$  above the critical temperature  $T_c$  of the electroweak phase transition in the Minimal Standard Model (MSM). In this scenario the number of particles can be slightly larger than the number of anti-particles implying non-vanishing fermion chemical potentials  $\mu_f \neq 0$ . We have calculated the  $T_c$  in the MSM for a thermal medium characterized by  $\mu_f \neq 0$ . The calculation was performed in the one-loop approximation to the effective potential at finite temperature, working in the real time formalism of the thermal field theory and using the Feynman gauge. To calculate the effective potential we have followed the procedure shown at [15].

We have calculated the  $T$  and  $\mu_f$  effects over the effective potential, and the Higgs boson condensate dependence on  $T$  and  $\mu_f$ . We have shown that at  $T = 0$  it exists a chemical potential critical value  $\mu_f^c$  for which the electroweak symmetry is restored. If  $T$  and  $\mu_f$  effects are simultaneously considered it was shown that for  $\mu_f \geq \mu_f^c$  then  $T_c^2 \leq 0$ , implying that the electroweak phase transition does not take place in this scenario. In the limit  $\mu_f = 0$  we have reproduced the  $T_c$  expression published in [15].

On the other hand we have shown for  $\mu_f < \mu_f^c$  that  $T_c(\mu_f) < T_c(0)$ , being  $T_c(0)$  the critical temperature of the electroweak phase transition for the case  $\mu_f = 0$ . The main non-vanishing fermion chemical potential effect over the electroweak phase transition



is to lower the  $T_c$  value respect to the case  $\mu_f = 0$ . We have assumed that  $\mu_{f_i} \neq 0$  for all the  $f_i$  quark and charged lepton flavours. The values of the different  $\mu_{f_i}$  are unknown and we argue that their values could be obtained through a mechanism of baryogenesis generating the current BAU at a  $T$  above  $T_c$ .

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## References

- [1] D. A. Kirzhnits, JETP Letters 15 (1972) 529; D. A. Kirzhnits and A. D. Linde, Phys. Lett B42 (1972) 471; Ann. Phys. 101 (1976) 195.
- [2] L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320.
- [3] S. Weinberg, Phys. Rev. D9 (1974) 3357.
- [4] A. D. Linde, Phys. Rev. D14 (1976) 3345.
- [5] A. D. Linde, Rep. Prog. Phys. 42 (1979) 389.
- [6] M. B. Gavela, P. Hernandez, J. Orloff and O. Pene, Mod. Phys. Lett. A9 (1994) 795; M. B. Gavela, P. Hernandez, J. Orloff, O. Pene and C. Quimbay, Nucl. Phys. B340 (1994) 382.
- [7] P. Huet and E. Sather, Phys. Rev. D51 (1995) 379.
- [8] D. Boyanovsky, H. J. de Vega, D. S. Lee, Y. J. Ng and S. Y. Wang, Phys. Rev. D59 (1999) 105001.
- [9] S. Y. Wang, D. Boyanovsky, H. J. de Vega, D. S. Lee and Y. J. Ng, Phys. Rev. D61 (2000) 065004.
- [10] L. M. Krauss and M. Trodden, Phys. Rev. Lett. 83 (1999) 1502.
- [11] For a review of possible mechanism for baryogenesis, see A. Dolgov, Phys. Rep. 222 (1992) 309.
- [12] A. Erdas, C. W. Kim and J. A. Lee, Phys. Rev. D48 (1993) 3901.
- [13] A.J. Niemi and G.W. Semenoff, Ann. Phys. (N.Y.) 152 (1984) 105.

- [14] N.P. Landsman and Ch.G. van Weert, Phys. Rep. 145 (1987) 141.
- [15] A. V. Smilga, Phys. Lett. B222 (1989) 462.
- [16] R. L. Kobes, G. W. Semenoff and N. Neiss, Z. Phys C29 (1985) 371.
- [17] J. I. Kapusta, Finite Temperature Field Theory. Cambridge University Press, 1989.
- [18] P. Bock et al., EP Working Group for Higgs boson searches, CERN-EP/2000-055. April 2000.

Figure 1: One-loop diagrams contributing to the effective potential at finite temperature.